# Markov Switching Models for Time Series Data with Dramatic Jumps (Model Peralihan Markov untuk Data Siri Masa dengan Lompatan Drastik)

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## ABSTRACT

In this research, the Markov switching autoregressive (MS-AR) model and six different time series modeling approaches are considered. These models are compared according to their performance for capturing the Iranian exchange rate series. The series has dramatic jump in early 2002 which coincides with the change in policy of the exchange rate regime. Our criteria are based on the AIC and BIC values. The results indicate that the MS-AR model can be considered as useful model, with the best fit, to evaluate the behaviors of Iran's exchange rate.

Keywords: Fluctuations of exchange rate; Markov Switching Autoregressive model; nonlinear times series models

# ABSTRAK

Dalam penyelidikan ini model autoregresi Markov (MS-AR) dan enam pendekatan model siri masa dipertimbangkan. Model-model ini dibandingkan mengikut keupayaan mendapatkan siri kadar pertukaran wang Iran. Siri ini mempunyai lompatan drastik pada awal 2002 yang berlaku serentak dengan perubahan polisi kadar regim pertukaran wang. Kriteria yang telah kami gunakan adalah berasaskan kepada nilai AIC dan BIC. Keputusan menujukkan bahawa model MS-AR boleh dikatakan berguna.

Kata kunci: Model autoregrasi peralihan Markov; model siri masa tak linear; naik-turun kadar pertukaran

## INTRODUCTION

Many economic time series associated with events such as financial crises, war or change in government money policy exhibit dramatic jumps in their behavior. When jumps arise in time series data, a powerful tool that up date themselves using a change in their regime is the Markov switching models. This model offers a better statistical fit to the data than the other models.

The ARCH model that was presented by Engle (1982) could be used in a large variety of modeling of the fluctuation of exchange rate (Kang, 1999; Kroner & Lastrapes, 1993; Wang & Wong, 1997). Engle and Hamilton (1990) used Markov switching (MS) model for survey fluctuations of dollar and showed that this model is better that the random walk model to forecast the fluctuation of dollars rate. Hamilton and Susmel (1994) pioneering work provides the evidence that a Markov switching autoregressive conditional heteroskedasticity (MS-ARCH) of exchange rates outperforms the ARCH and GARCH models for the New York stock exchange. Bollen et al. (2000) showed that the Markov switching model captures the dynamics of exchange rates better than the alternative time series models. Lee and Chen (2006) discussed the Markov switching model in exchange rate prediction. Ismail and Isa (2006) showed that the MS-AR

model is the best-fitted model for modeling fluctuations of exchange rates for three Asian countries.

To control their currencies many developed countries have used the managed floating of exchange rates regime since the mid-1980s. In early 2002, the Iranian government adopted a managed floating for administering of the fluctuations of the exchange rate. This change in policy for exchange rate regime in Iran caused dramatic increase in exchange rate of the Rial per dollar (Figure 1). To find the best-fitted model for the behaviors of Iran's exchange rate after introducing the data, two stages were taken. In the first stage, the comparison of the AR, ARMA, ARCH and GARCH models using the model selection criteria (AIC) will be given. In the second stage, after introducing the nonlinear additive AR, self-exciting threshold AR, logistic smooth transition AR, and Markov switching AR models briefly, they will be compared for analysis of our data using AIC, BIC values.

# DATA AND MODEL SELECTION

In this study, we employed the Iran's Rial per the U.S. dollar collected monthly for the period 1995-2009 by the International Monetary Fund (IMF) and could be obtained from http://www.imf.org . The variable under investigation

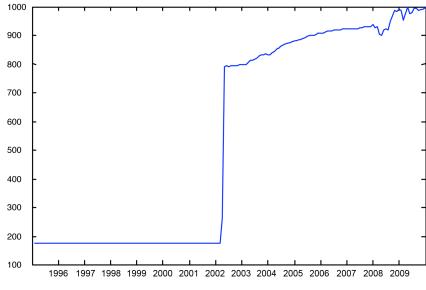


FIGURE 1. The Exchange rate series of the Iranian Rial per the US dollar

is exchange rate returns in percentage  $y_t = 100 \text{ x} [\text{In}(r_t) - \text{In}(r_{t-1})]$  where  $r_t$  is the monthly exchange rate. Figure 1 shows the Exchange rate series of the Iranian Rial per the US dollar.

To identify the best fitted model among several linear and nonlinear time series models, we use the Akaike information criterion (AIC) (Akaike 1974) or Bayesian Information Criterion (BIC) (Akaike 1979). These criteria measure the deviation of the fitted model from the actual one. The model with the minimum value of AIC or BIC is chosen. In this article, we compared eight different time series models based on these criteria.

#### COMPARISON OF AR, ARMA, ARCH, AND GARCH MODELS

In this section, we considered the AR (k), ARMA (k, l) (Priestley 1988), ARCH (q), and GARCH (p,q) (Mills & Markellos 2008) time series models. To find the order of fitted AR and ARMA models the plots of autocorrelation function and AIC are used. We used likelihood ratio test in order to discriminate between a p-lag and a q-lag ARCH and GARCH processes. The estimated parameters for the best-fitted models i.e. AR (2), ARMA (2, 1), ARCH (2), and GARCH (1, 2) models are shown in Table 1. The estimated parameters for AR model are more significant than ARCH and GARCH models. The estimated parameters for ARCH and GARCH models are spurious; consequently, this is evidence for the rejection of these models for time series modeling with dramatic jumps. On the other hand, the value of AIC for AR model is less than for the others; therefore, the AR model will be extended to the nonlinear autoregressive models such as NAAR, SETAR, LSTAR, and MS-AR in the following sections.

# NONLINEAR ADDITIVE AUTOREGRESSIVE MODEL

A time series  $y_t$  follows a nonlinear additive autoregressive (NAAR) model if:

$$y_{t} = f_{0}\left(t\right) + \sum_{i=1}^{p} f_{f}\left(y_{t-i}\right) + \varepsilon_{t}, \qquad (1)$$

where the  $\varepsilon_i$  is noise series and  $f_i(.)$  are univariate smooth functions. Function  $f_i(.)$  is represented by penalized cubic regression splines and estimated by quadratically penalized likelihood maximization. (Wood 2004, 2006 and 2011).

### SELF-EXCITING THRESHOLD AUTOREGRESSIVE MODEL

The threshold autoregressive (TAR) model introduced by Tong (1990). A 2-regime TAR model is given by:

$$y_{t} = \left( \left( a_{0,0} + \sum_{i=1}^{p} a_{i,0} y_{t-i} \right) \left( 1 - 1 \left[ z_{i} > r \right] \right) \right), \tag{2}$$

where  $\varepsilon_t s$  are iid  $\left(0, \sigma \frac{2}{\epsilon}\right)$ . It is also called the Self-Exciting

Threshold Autoregressive (SETAR) when the threshold variable  $z_t$  is taken to be a lagged value of the time series itself that is  $z_t = y_{t-d}$ . in (2) the threshold value r is estimated by selecting the best fitted model. The SETAR model assumes that the regime  $S_t$  is determined by value I [Z<sub>t</sub>>r]. So, define  $S_t$  such that:

$$S_t = \begin{cases} 0 & z_t \le r \\ 1 & z_t > r \end{cases}$$

Then,  $S_t$  follows a first-order Markov process with transition matrix  $\xi$  given by:

$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{\Phi}(r_1) & 1 - \boldsymbol{\Phi}(r_1) \\ \boldsymbol{\Phi}(r_2) & 1 - \boldsymbol{\Phi}(r_2) \end{pmatrix},$$

		Coefficient	Stand. Error
AR	a <sub>0</sub> (constant)	0.7607	0.6294(0.2284)
	a <sub>1</sub>	0.3565	0.0748(3.7e <sup>-0.6</sup> )***
	a <sub>2</sub>	-0.128	0.0886(0.0886)
	σ AIC	8.2255 760.38	
	a <sub>o</sub> (Constant)	0.9989	0.9009(0.268)
ARMA	a <sub>1</sub>	-0.021	0.4975(0.997)
	a <sub>2</sub>	-0.0138	0.1938(0.943)
	b <sub>1</sub>	0.652	0.4929(0.459)
	σ AIC	8.2861 1272.99	
ARCH	$\alpha_0$ (Constant)	69.17	5.255(<2e <sup>-16</sup> )***
	$\alpha_{_1}$	1.051	0.376(0.052)
	$\begin{array}{c} \alpha_2 \\ AIC \end{array}$	1.09e <sup>-13</sup> 1122.35	0.0114(1.00)
	$\alpha_0$ (Constant)	65.33	0.6294(0.0002)***
GARCH	$\alpha_{_1}$	0.1858	$0.0748(<2e^{-16})^{***}$
	$\alpha_2$	0.415	0.1062(0.6958)
	$\underset{AIC}{\beta_1}$	8.0e <sup>-13</sup> 1139.78	0.2604(1.00)

TABLE 1. The estimated parameters of AR, ARMA, ARCH, and GARCH model

P-values are reported in the parenthesis .\*\*\*,\*\*,\* denotes significance of the coefficient at the 0%, 10%, 5% level.

with  $\Phi$  (.) being the standard normal distribution, and  $rk = (r-\mu_i)/\sigma$  The conditional on  $Z_i$ >r the model converts to an AR model. So, this model is estimated for a fixed threshold value.

### LOGISTIC SMOOTH TRANSITION AUTOREGRESSIVE MODEL

The Logistic Smooth Transition Autoregressive (LSTAR) can be viewed as a generalization of the SETAR model. A more gradual transition between the different regimes can be obtained by replacing the indicator function I  $[Z_t>r]$  in (2) by a continuous function logistic  $G(zt;\gamma r)$  given by:

$$G(z_{t}, \gamma, r) = \frac{1}{1 + \exp(-\gamma [z_t - r]]}.$$
(3)

The resultant model is called a Logistic Smooth Transition Autoregressive (LSTAR). The parameter r in (3) can be interpreted as the threshold between the two regimes corresponding to  $G(z_t; \gamma, r)=0$  and  $G(z_t; \gamma, r)=1$ , in the sense that the logistic function changes monotonically from 0 to 1 as  $z_t$  increases, while  $G(z_t; \gamma, r)=0.5$ . The parameter  $\gamma$  determines the smoothness of the change in the value of the logistic function (Teräsvirta 1994; Franses & van Dijk 2000).

#### MARKOV SWITCHING AUTOREGRESSIVE MODEL

The Markov switching autoregressive (MS-AR) model introduced by Hamilton (1989). MS-AR model with two regimes is written as:

$$y_{t} = a_{o,st} + a_{1,st}y_{t-1} + \dots + a_{p,st}y_{t-p} + \in_{t}$$
(4)  
$$\in_{t} \sim NID(0, \sigma_{s_{t}}^{2})$$
$$a_{i,st} = a_{i1}(1 - s_{t}) + a_{i2}s_{t} , i = 1, \dots, p$$
$$\sigma_{s_{t}}^{2} = \sigma_{1}^{2}(1 - s_{t}) + \sigma_{2}^{2}s_{t}$$
$$s_{t} = 0,1 \quad (\text{Re gime } 0, 1).$$

For  $t=1,\ldots,T$ .

In this model, the parameters are depended on the regime at time t, indexed by st, that regimes are discrete unobservable variable. The transition between the regimes is governed by a first order Markov process as follows:

$$p_{ij} = \Pr(s_i = j | s_{i-1} = i) \quad \forall i, j = 0, 1, \sum_{j=0}^{i} p_{ij} = 1.$$

It is convenient to summarize these transition probabilities in p is written as:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix}.$$
 (5)

With  $p_{00}+p_{01}=1$ ,  $p_{10}+p_{11}=1$ .

To estimate the parameters of MS-AR model by MLE the density of yt given the past information of  $\Psi_{i-1}$  is:

$$f(y_{t}|\Psi_{t-1},s_{t}) = \frac{1}{\sqrt{2\pi\sigma_{s_{t}}^{2}}} \exp\left(\frac{\left\{y_{t} - \left(a_{0,st} + a_{1,s_{t}},y_{t-1} + \ldots + a_{p,st},y_{t-p}\right)\right\}^{2}}{2\sigma_{s_{t}}^{2}}\right)$$

We need the values of st which are unobserved. Thus, we have:

$$\begin{split} &f\left(\boldsymbol{y}_{t} \left| \boldsymbol{\Psi}_{t-1} \right. \right) = \sum_{\boldsymbol{s}_{t}=0}^{1} f\left(\boldsymbol{y}_{t}, \boldsymbol{s}_{t} \left| \boldsymbol{\Psi}_{t-1} \right. \right) \\ &= \sum_{t} f\left(\boldsymbol{y}_{t} \left| \boldsymbol{s}_{t}, \boldsymbol{\Psi}_{t-1} \right. \right) P\left[\boldsymbol{s}_{t} \left| \boldsymbol{\Psi}_{t-1} \right. \right]. \end{split}$$

Then, the log likelihood function is given by:

In L = 
$$\sum_{t=1}^{T} \ln \left\{ \sum_{s_t=0}^{1} f(y_t | s_t, \Psi_{t-1}) P[s_t | \Psi_{t-1}] \right\}.$$
 (6)

In (6),  $P[s_t \Psi_{t-1}]$  is filtered probabilities, which this probabilities calculate using of the Filter introduced by Hamilton (1989) for t=1,..., T. Filtered probabilities refer to inferences about  $s_t$  conditional on information up to t,  $\Psi_t$ . The next step, we calculate smoothed probabilities  $P[s_t|\Psi_T]$  using all the information in the sample,  $\Psi_T$  for t= T-1, T-2, ..., 1 and given  $P[s_t|\Psi_T]$  at the last iteration of filter.

# COMPARISON OF NAAR, SETAR, LSTAR, AND MS-AR MODELS

In this section we first extended the AR (2) process NAAR (2) model. The estimated parameters for NAAR model is shown in Table 2. The values of AIC and BIC for this model is less than that obtained for AR model. Thus, at this stage the NAAR model is preferred.

Now as we have the change in policy of the exchange rate regime of Iran in 2002, therefore it seems that the nonlinear autoregressive models with switching regime such as SETAR, LSTAR, and MSAR are more appropriate than NAAR model.

Then, we estimated SETAR, LSTAR, and MSAR. In these models, the parameters were allowed to shift in the various regimes. For MSAR models the transition between regimes were only depended on previous regime, as it follows a hidden Markov chain with transition probabilities. Also in this model regimes were unobserved, thus transition between regimes is stochastic. But, in a SETAR and LSTAR models transition between regimes depend to the past observation of the process, since the transition matrix and regimes are known, thus transition between regimes was deterministic.

We determined the number of regimes using AIC (Cologni & Manera 2009; Psaradakis & Spagnolo

TABLE 2. The estimated parameters of NAAR model.

	Coefficient	Stand. Error	T.value	Pr(>ltl)		
f <sub>0</sub>	0.9849	0.2574	3.8262	0.00183***		
	Approximate significance of smooth terms					
	edf	Ref.df	F	p-value		
$f_1$	5.222	5.7278	172.962	< 2e <sup>-16 ***</sup>		
$f_2$	2.9492	3.2901	2.3436	0.06908*		
		AIC BIC	467.1829 527.7433			

P-values are reported in the parenthesis .<sup>\*\*\*</sup>,<sup>\*\*</sup>, \* denotes Significance of the coefficient at the 0%, 1%, 5% level. edf is the array of estimated degrees of freedom for the model terms.

2003). Using this strategy, we considered two-regime for modelling nonlinear autoregressive models with switching regime. Regimes 0 and 1 describe low and high the fluctuations of exchange rate phases respectively. Note that when the fluctuations of exchange rate are low, the process is in low regime. On the other hand, when the fluctuations of exchange rate are high (and increase); the process is in high regime.

TABLE 3. The estimated parameters of SETAR model with details.

		Coefficient	Stand. Error	
	$a_0$	0.3824	0.3616 (0.2918)	
	$a_1$	-0.288	0.8008 (0.7195)	
Regime 0	$a_2$	-0.007	0.0401 (0.8602)	
	σ	4.3278		
	a <sub>o</sub>	0.5950	0.8895(0.5044)	
Regime 1	a <sub>1</sub>	2.3124	$0.1021$ ) <2 $e^{-16^{***}}$ (	
	$a_2$	-6.1444	0.2873(<2e <sup>-16</sup> )***	
	σ	4.3278		
threshold value			0.4059	
Proportion of points in regime 0			84.75%	
Test of Linear against SETAR			1908.462 (0.00)***	
AIC 538.5391				
BIC 560.8508				

P-values are reported in the parenthesis .<sup>\*\*\*</sup>,<sup>\*\*</sup>, <sup>\*\*</sup> denotes Significance of the coefficient at the 0%, 1%, 5% level.

Empirical details for the SETAR model are presented in Table 3. Test of linearity against threshold (Hansen 1999) is statistically significant. Thus, the AR process is rejected against SETAR model; see Table 3. In order to determine Lagged value of threshold variable  $z_i$ , the AIC is used and the threshold variable in the form of  $z_i = y_{i-1}$  is selected. Thus, the fitted model is as follows (Table 3):

$$y_{t} = (0.3824 - 0.288 y_{t-1} - 0.007 y_{t-2})(1 - I[y_{t-1} > 0.4059]) + (0.595 + 2.3124 y_{t-1} - 6.1444 y_{t-2}(I[y_{t-1} > 0.4059]).$$

Note that the two-lag autoregressive for second regime are statistically significant at conventional significance levels. Proportion of points in regime 0 is 84.75% (Table 3) which indicates that the process is more in regime 0. In addition, the values of AIC and BIC for this model are 538.5391 and 560.8508, respectively.

The fitted LSTAR model is:

$$y_{t} = (0.3817 - 0.1675y_{t-1} - 0.0073 y_{t-2})(1 - G[y_{t-1} > 3.41)) + (7145.6626 - 172.8318 y_{t-1} - 292.8142 y_{t-2} (G[y_{t-1} > 3.41)),$$

where:

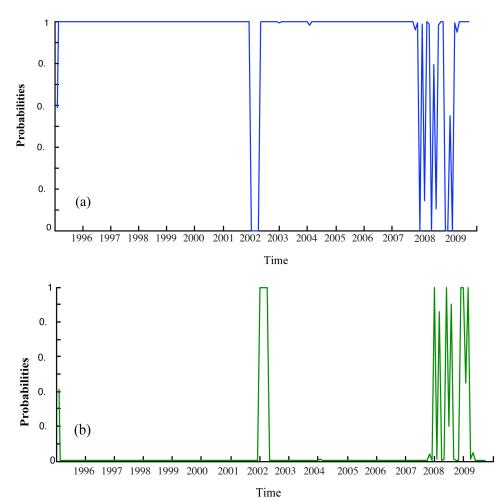
$$G(z_{t};\gamma,r) = \frac{1}{1 + \exp(-39.99[y_{t-1} - 3.14])}$$

The values of AIC and BIC for LSTAR model are 418.670 and 444.8362, respectively which are less than that obtain a for the SETAR model. Therefore, when a more gradual transition occurs between different regimes, then it seems that the SETAR model is more suitable than the others.

TABLE 4. The estimated paremeters of MS-AR model with details.

model with details.				
	Coefficient	Stand. Error		
a <sub>o</sub>	0.1099	0.0152 (0.00)***		
$\mathbf{a}_1$	0.2521	0.0186 (0.00)***		
$a_2$	-0.0488	0.0056 (0.00)***		
σ	0.0148			
a <sub>o</sub>	34.1187	1.1170 (0.00)***		
$\mathbf{a}_1$	0.0548	0.0015(0.00)***		
a <sub>2</sub>	-0.4663	0.0294 (0.00)***		
σ	14.5143			
Regime 0	Regime	1		
Regime 0 0.91				
0.09	0.37			
the expected duration of regime 0 11.59				
duration o	1.58			
ood	-161.7751			
AIC				
BIC				
	$\begin{array}{c} a_{o} \\ a_{1} \\ a_{2} \\ \sigma \\ \end{array}$ $\begin{array}{c} \sigma \\ a_{o} \\ a_{1} \\ a_{2} \\ \sigma \\ \end{array}$ $\begin{array}{c} \sigma \\ \end{array}$ $\begin{array}{c} Regime \ 0 \\ 0.91 \\ 0.09 \\ \end{array}$ $\begin{array}{c} duration \ o \\ duration \ o \end{array}$	$\begin{tabular}{ c c c c } \hline Coefficient \\ \hline a_o & 0.1099 \\ \hline a_1 & 0.2521 \\ \hline a_2 & -0.0488 \\ \hline \sigma & 0.0148 \\ \hline a_o & 34.1187 \\ \hline a_1 & 0.0548 \\ \hline a_2 & -0.4663 \\ \hline \sigma & 14.5143 \\ \hline Regime 0 & Regime \\ 0.91 & 0.63 \\ 0.09 & 0.37 \\ \hline duration of regime 0 \\ \hline duration of regime 1 \\ \hline \end{tabular}$		

P-values are reported in the parenthesis .\*\*\*,\*\* ,\* denotes Significance of the coefficient at the 0%, 1%, 5% level.



RAJAH 2. Smoothed probabilities of regimes (a) Probabilities of Regime 0 and (b) Probabilities of Regime 1

Using the criteria: values of AIC and BIC, value of the log-likelihood function, estimated matrix of transition probabilities that showed the probability of switching between regimes, estimated variance and considering the significant of P-value of estimated coefficients, we compared the different types of Markov switching autoregressive models. Using this selection strategy, the best performance was obtained for the MS-AR model with two regimes with a two-lag autoregressive components. The details of the model fitted for MS-AR is presented in Table 4. All estimated coefficients are statistically significant at conventional significance levels. The transition probabilities  $(p(s_t = 0 | s_{t-1} = 0) = 0.91)$  and  $(p(s_t = 1 | s_{t-1} = 1) = 0.37)$  suggest that the regime 0 is found to be more persistent. When the process is in regime 0, there is a low probability that it switches to regime  $p(s_t = 1 | s_{t-1} = 0) = 0.09)$ . The average duration of each regime supports this conclusion, as can be seen from Table 4, the expected duration of regime 0 and 1 are 11.59 and 1.58, respectively.

Figure 2 shows the time series of smoothed probabilities for fluctuations of the exchange rate of Iran based on the MSAR model. This figure shows the probability of being in regime 0 or 1 at specific time. At the beginning of 2002 the fluctuations for exchange rate of Iran was extremely high which causes the process be in regime 1 with probability 1. In the other years since the fluctuations for exchange rate is low, therefore the process is in regime 0 with a high probability. In 2008 and 2009 there were some fluctuations at exchange rate which causes transition from one regime to the other alternatively.

The deviation of the fitted model from MSAR based on the values of AIC and BIC are 347.55 and 385.66 respectively.

Table 5 shows the comparison of the AIC for different time series models such as AR, ARMA, ARCH, GARCH, NAAR, SETAR, LSTAR, MS-AR. The results show that among these models the MSAR model is best fit for modeling the fluctuations of the exchange rate of Iran, with the lowest AIC value.

TABLE 5. The values of AIC for different methods		
Model	AIC	
AR	760.38	
ARMA	1272.99	
ARCH	1122.35	
GARCH	1139.78	
NAAR	467.18	
SETAR	538.24	
LSTAR	418.67	
MSAR	347.55	

# CONCLUSION

The MSAR model has the least value of AIC comparing the other models for the modeling of the Iranian exchange rate series fluctuations. In addition, using the MSAR model enables us to have better estimated parameters that are statistically significant at conventional significance levels. The plots of the smoothed probabilities of regimes are obviously showing the fluctuations of the Iranian exchange rate series.

Consequently, the MSAR model can be considered as a powerful tool for modeling time series with dramatic jumps in their behavior. It should be noted that there is an alternative method, namely, Singular Spectrum Analysis (SSA) which works very well for time series analysis, forecasting and change point detection (Hassani 2007; Hassani et al. 2009; Hassani et al. 2010; Hassani & Thomakos 2010). Applying the SSA method is part of our future plan.

#### REFERENCES

- Akaike, H. 1974. A new look at statistical model identification, IEEE Transactions on Automatic Control 19: 716-723.
- Akaike, H. 1979. A Bayesian extension of the minimum AIC procedure. *Biometrika* 66: 237-242.
- Bollen, NP. B., Gray, S.F. & Whaley, R.E. 2000. Regime switching inforeign exchange rates: Evidence from currency option prices. *Journal of Econometrics* 94: 239-276.
- Cologni, A. & Manera, M. 2009. The asymmetric effects of oil shocks on output growth: A Markov–Switching analysis for the G-7 countries. *Economic Modelling* 26: 1-29.
- Engle, R.F. 1982. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. In\_ation, *Econometrica*, 50: 987-1007.
- Engel, C. & Hamilton, J.D. 1990. Long switching in the dollar: are they the data and do Markets know it? *American Economic Review* 80: 689-713.
- Franses, P.H. & Dijk, D.V. 2000. Non-linear time series models in empirical\_nance, Cambridge: Cambridge University Press.
- Hamilton, J.D. 1989. A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica* 57: 357-384.
- Hamilton, J.D. & Susmel, R. 1994. Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics* 64: 307-333.
- Hansen, B. 1999. Testing for linearity. Journal of Economic Surveys 13(5): 551-576.
- Hassani, H. & Thomakos, D. 2010. A Review on Singular Spectrum Analysis for Economic and Financial Time Series, *Statistics and its Interface* 3(3): 377-397.
- Hassani, H., Heravi, H. & Zhigljavsky, A. 2009. Forecasting European Industrial Production with Singular Spectrum Analysis, *International Journal of Forecasting* 25(1): 103-118.
- Hassani, H. 2007. Singular Spectrum Analysis: Methodology and Comparison. *Journal of Data Science* 5(2): 239-257.

- Hassani, H., Dionisio, A. & Ghodsi, M. 2010. The effect of noise reduction in measuring the linear and nonlinear dependency of financial markets, Nonlinear Analysis: *Real World Applications* 11(1): 492-502.
- Ismail, M.T. & Isa, Z. 2006. Modelling Exchange Rates Using Regime Switching Models. *Sains Malaysiana* 35(2): 55-62.
- Kang, I.B. 1999. International foreign exchange agreements and nominal exchange rate volatility: a GARCH application. *The North American Journal of Economics and Finance*, 10(2): 453-472.
- Kroner, K.F. & Lastrapes, W.D. 1993. The impact of exchange rate volatility on international trade:Reduced form estimates using the GARCH-in-mean model. *Journal of International Money and Finance* 12(3): 298-318.
- Lee, Y.H. & Chen, L.S. 2006. Why use Markov-switching models in exchange rate prediction? *Economic Modelling*, 23: 662-668.
- Mills, C.T. & Markellos, N.R. 2008. The Econometric Modelling of Financial Time Series. Cambridge University Press.
- Priestley, M.B. 1988. Non-linear and Non-stationary Time Series Analysis. NY: Academic Press INC.
- Psaradakis, Z. & Spagnolo, N. 2003. On the determination of the number of regimes in Markov–Switching autoregressive models, *Journal of Time Series Analysis* 24: 237-252.
- Teräsvirta, T. 1994. Specification, estimation, and evaluation of smooth transition autoregressive models, *Journal of the American Statistical Association* 89: 208-18.

- Tong, H. 1990. Non-Linear Time Series: A Dynamical Systems Approach. Oxford: Oxford University Press.
- Wang, J.X. & Wong, H.I. 1997. The predictability of Asian exchange rates: evidence from Kalman filter and ARCH estimations. *Journal of Multinational Financial Management*, 7(3): 231-252.
- Wood, S.N. 2004. Stable and efficient multiple smoothing parameter estimation for generalized additive models. *J. Amer. Statist.* 99:673-686.
- Wood, S.N. 2006. *Generalized Additive Models: An Introduction with R*, NY: CRC. Press
- Wood, S.N. 2011. Fast Press stable restricted maximum likelihood and marginal likelihood estimation of semi parametric generalized linear models. *Journal of the Royal Statistical Society (B)* 73(1): 3-36.

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